Static models of the Edgeworth cycle*

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Abstract

We provide a reduced form model that encompasses a range of static explanations for the Edgeworth cycle. The model distills two common features that lie behind a range of intuitive explanations for Edgeworth cycles: discontinuity in demand and a positive residual demand.
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Regular, asymmetric cycles in prices, known as Edgeworth cycles, are observed in a number of retail petrol markets including cities in Australia, Canada, Norway, and the United States. There are a range of contending theories of the cycle, including the original Edgeworth (1925) model and the popular Maskin and Tirole (1988) model. We provide a reduced form synthesis of a class of static models of the Edgeworth cycle. This class is appealing for its simplicity and the range of intuitive explanations for the cycle encompassed, but suffers from the limitation that an essentially static model is employed to explain a dynamic pricing pattern. We make this limitation explicit through the use of Assumption 3 below, in which we impose play of myopic best responses.

A theory of Edgeworth pricing must explain the incentive to *undercut* rivals when rival prices are "high" and the incentive to *relent* and raise price when rival prices are "low". The Edgeworth (1925) model provides these incentives in an environment of capacity constrained price competition for a homogeneous product: homogeneous products give rise to a discontinuity in demand which provides the incentive to undercut, while the capacity constraint of one's rival gives rise to residual demand, providing the incentive to relent when the price of one's rival is low. The game below admits the Edgeworth model as a special case.

In the game G, two firms compete by simultaneously setting price over an infinite horizon. We omit costs for exposition, and we omit time subscripts for notational convenience. Define the demand functions $d_i(p)$, $i = 1,2,3$, and the associated revenue functions $g_i(p) = pd_i(p)$, $i = 1,2,3$. We can think of $d_1(p)$ as a firm's demand curve when undercutting its rival,

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$d_2(p)$ as the demand from matching its rival's price, and $d_3(p)$ as the firm's residual demand when its rival undercuts. Define the prices $p^m \equiv \arg\max_p g_1(p)$ and $p^r \equiv \arg\max_p g_3(p)$, and define the subset of the pricing space $\mathscr{P} = [0, p^m]$. We can interpret p^m and p^r as the monopoly prices with respect to market demand and residual demand, respectively.

Given the price vector $\{p_i, p_j\}$, let the (residual) demand for firm i be

$$q_i(p_i, p_j) = \begin{cases} d_1(p_i) & \text{if} \quad p_i < p_j \\ d_2(p_i) & \text{if} \quad p_i = p_j \\ d_3(p_i) & \text{if} \quad p_i > p_j, \end{cases} \tag{1}$$

where p_i and p_j are the prices of firm i and firm j, respectively. We impose the following assumptions.

Assumption 1. *The demand functions $d_i(p), i = 1, 2, 3$, are strictly decreasing, continuously differentiable, and intersect both axes. $\forall p \in \mathscr{P}, d_1(p) > d_2(p) > d_3(p) > 0$.*

Assumption 2. *For $i = 1, 2, 3$, $g_i(p)$ is strictly concave on the domain $\mathscr{P}$.*

Assumption 3. *Firms play a myopic best response to their rival's previous price.*

The concavity of $g_1(p)$ ensures that $g_1(p)$ is increasing on $\mathscr{P}$. Therefore, if firm i *undercuts* its rival's price, p_j, it will do so infinitesimally and profits will be approximated by $g_1(p_j)$. Assumption 1 then guarantees that undercutting is always preferred to price matching. If firm i raises its price above p_j, it will *relent* optimally to p^r, obtaining payoff $g_3(p^r)$. The following proposition suggests that firm i undercuts if p_j is sufficiently high, and relents otherwise.

Proposition 1. *There exists $p^* \in \mathscr{P}$ such that the best response for firm i to p_j is to marginally undercut if $p_j > p^*$ and relent to p^r if $p_j \leq p^*$.*

Proof. Define $h(p_j) = g_1(p_j) - g_3(p^r)$. The result follows from the following 3 observations: (i) $h(0) < 0$. Notice that $g_1(0) = 0$, and by Assumption 1, $g_3(p^r) > 0$. (ii) $h(p^m) > 0$. By Assumption 1, $g_1(p) > g_3(p) \forall p \in \mathscr{P}$. This implies that $g_1(p^m) > g_1(p^r) > g_3(p^r)$. (iii) $h'(p) > 0 \forall p \in \mathscr{P}$. This follows because $g_1(p)$ is increasing on $\mathscr{P}$ and $g_3(p^r)$ is independent of p_j. □

The intuition is straightforward. The discontinuity in residual demand provides a strong incentive to undercut when your rival's price is high. The existence of residual demand when your price is above your rival's provides an incentive to relent when your rival's price is low.

It is a small step to transform the game G into an Edgeworth cycle model. In particular, if we restrict the price space to a discrete grid, Proposition 1 suggests an asymmetric price cycle, with firms lowering prices gradually (subject to the coarseness of the price grid), and relenting more dramatically to p^r.

Notice that Assumptions 1 - 3, together with the residual demand function, (1), are sufficient, but more restrictive than necessary for Proposition 1. As specified, the game G encompasses the Edgeworth model under efficient rationing for suitable choice of capacity constraints. We need only specify the residual demand relation (1). Notice that it also permits a range of other motivations. For example, consider a stylised model of heterogeneous consumer search costs in a market for a homogeneous product. Suppose a subset of consumers are perfectly informed about the set of available prices, while the remaining consumers face prohibitive search costs, considering only their local firm. The existence of searching consumers provides the discontinuity in residual demand, while the non-searching consumers provide positive residual demand for a relenting firm. See Robertson (2008) for additional discussion.

Some product differentiation settings can also provide a motivation for Edgeworth cycles. For example, if we introduce a mass point of consumers into the Hotelling (1929) linear city model, we will satisfy the discontinuity of demand and positive residual demand of the above framework.[1] Relatedly, Soetevent (2011) generalises the Hotelling model from competition on a line to competition on graphs. For a range of such graphs, discontinuity in demand and a positive residual demand are also present. With a slight modification to the game G, we can also accommodate this model for suitable parameterisations.[2]

To sum up, the game G is sufficiently general to encompass capacity constrained price competition, stylised settings of consumer search cost, and some product differentiation settings. Given that the framework requires only a discontinuity in residual demand, and a positive residual demand above the discontinuity, we imagine there are other settings that would conform also. The primary limitation of the game as an explanation of the Edgeworth cycle is that, as Assumption 3 makes explicit, it is essentially a static model invoked to explain a dynamic setting.

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[1] To see this, consider the most basic setting. Consider the linear city with consumers of measure 1 uniformly distributed on the interval $[0,1]$. Suppose firms are located at the endpoints of the city, and transport costs are linear or quadratic in distance. In addition, suppose there is a mass point of consumers in the middle of the city. It is easy to see that this setting is encompassed by the game G. For slightly more general applications of the linear city model, we would need to slightly modify equation (1), replacing the point of discontinuity at p_j with an alternative at some $\hat{p}(p_j)$.

[2] Again, we would need to modify equation (1), replacing the discontinuity point p_j with an alternative at some $\hat{p}(p_j)$.