Edgeworth cycles with partial price commitment*

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Abstract

The price commitment model of Maskin and Tirole (1988) provides an extensively cited foundation for Edgeworth cycles. We examine the viability of Edgeworth cycles when price commitment is partial in the sense that a subset of firms are committed to price in each period. If multiple firms are not committed in each period, then the existence of Edgeworth cycle equilibria requires a demanding convexity condition on the profit function.

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At least since Castanias and Johnson (1993), the Maskin and Tirole (1988) (hereafter, MT) theory of price commitment has been invoked to explain asymmetric cycles in prices known as Edgeworth cycles. Under MT, two firms produce identical products and alternate in choosing prices from a discrete price grid. Equilibria exhibiting Edgeworth cycles involve two phases. Firms marginally *undercut* their rival when their rival is committed to a high price. At low prices, the incentive to undercut dissipates, and firms have an incentive to *relent* by raising price. The result is a highly asymmetric cycle in which prices fall gradually and then are rapidly restored.

Sequential timing plays a critical role in MT by allowing a firm to marginally undercut, confident that its rival is committed to price.[1] However, the ideal of strictly sequential timing is commonly violated in oligopoly. In the time taken to collect market data, assess the

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[1] In computationally extending the MT model to three firms, Noel (2008) maintains strictly sequential timing. Each firm adjusts price every three periods.

situation, decide on a course of action, and implement a change, rivals may have an opportunity to act.[2] We extend the theory to admit partial price commitment in the sense that a subset of rivals may be committed to price when a firm adjusts price. If multiple firms are not committed in each period, then the existence of Edgeworth cycle equilibria requires a demanding convexity condition on the profit function.

1 Partial price commitment

Over an infinite horizon, n firms compete for a homogeneous product by choosing prices from a discrete price grid. Firms discount the future at the common rate δ. We consider two separate timing protocols. Timing is deteriminstic in Section 1.1 and stochastic in Section 1.2. We first describe the profit function, which is common to both settings.

Given a price p_j and a market demand function D, industry profits are given by

$$\pi(p_j) = (p_j - c)D(p_j).$$

We normalise marginal costs to zero, $c = 0$. In each period, the market is shared equally between all firms charging the lowest price. Given a price vector $p = (p_1, \ldots, p_n)$ with lowest price p_j, if m firms set the price p_j, then profits for firm i are given by

$$\pi_i(p) = \begin{cases} \pi(p_j)/m & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases}$$

1.1 Deterministic timing

In each period t, each firm in the non-empty set J_t has the opportunity to adjust price, while the remaining $n - |J_t|$ firms must wait for this privilege. Every firm can adjust price every T periods, and hence $J_t = J_{t+kT}$, for any integer k. The ability of each firm to set price knowing their rival is committed to her price plays a key role in the MT model. This special case applies when $n = T = 2$ and $|J_t| = 1$ for all t. If $n > T$, then there are periods in which multiple firms simultaneously set price.

Like MT, we restrict attention to Markov strategies in which firms condition only on payoff-relevant states. In period t, the prices committed by rivals in previous periods, $p_{j \notin J_t}$,

[2] For multi-site firms with centralised decision making, the implementation lag alone can be lengthy. For example, Wang (2009) observes that in the retail petrol market of Perth in 2000, between 11am and 1pm, "48 of 73 BP sites hiked price to exactly 92.9 cents", with 13 of these changes occurring between 11am and noon. BP is a major retailer of petrol in this market, and controls a network of retail sites. This statement implies that, for the majority of the BP sites, it took more than an hour to implement a price change.

are payoff relevant. We summarise the dynamic problem faced by firm $i \in J_t$ when contemplating her period t choice of price via the Bellman equations

$$V_i^0(p_{j\notin J_t}) = \max_{p_i} \mathbb{E}_{p_{j\in J_t\setminus i}} \left(\pi_i(p) + \delta V_i^{T-1}(p_{j\notin J_{t+1}})\right), \tag{1}$$

$$V_i^\tau(p_{j\notin J_{t-\tau}}) = \mathbb{E}_{p_{j\in J_{t-\tau}}} \left(\pi_i(p) + \delta V_i^{\tau-1}(p_{j\notin J_{t-\tau+1}})\right),\ \tau = 1,\dots,T-1, \tag{2}$$

where expectations are taken over the (possibly mixed) strategies of rivals. $V_i^0(p_{j\notin J_t})$ is the value of firm i when it is her turn to choose price, given the vector of committed prices $p_{j\notin J_t}$. Her price p_i influences her profits today and her continuation value V_i^{T-1}. V_i^τ is the value of firm i when she has to wait τ periods for the next opportunity to adjust price.

We examine the viability of Markov perfect equilibria (MPE) exhibiting Edgeworth cycles. Given prices $p^1 > p^2 > \cdots > p^k$ with $k \geq T+2$, consider symmetric strategies of the form

$$R_i^D(p_{j\notin J_t}) = \begin{cases} p^{s+1} & \text{if } \min_{j\notin J_t} p_j = p^s,\ s = 1,\dots,k-1, \\ \left.\begin{array}{ll} p^1 & \text{with probability } \mu_i(p_{j\notin J_t}) \\ p^k & \text{with probability } 1-\mu_i(p_{j\notin J_t}) \end{array}\right\} & \text{if } \min_{j\notin J_t} p_j = p^k. \end{cases} \tag{3}$$

We use the shorthand p^s to refer to states in which $\min_{j\notin J_t} p_j = p^s$, for $s = 1,\dots,k$. The reaction functions R^D incorporate the essential elements of an Edgeworth cycle. In the undercutting phase, prices follow a gradual downward trajectory, and in the relenting phase, prices jump to the top of the cycle in a single step. Firms adopt mixed behavioural strategies at the cycle trough, p^k. If she moves at the trough, firm i raises price to p^1 with probability $\mu_i(p_{j\notin J_t})$, and sets $p_i = p^k$ otherwise. This results in a war of attrition at the trough as in MT.[3]

The strategies in (3) place no restrictions on the reaction functions for state vectors that are off the equilibrium path. They also contain, as a special case, the strategies employed by MT in their constructive proof of the existence of Edgeworth cycle equilibria in the two firm problem. In particular, under the MT strategies, $p^k = 0$, p^1 is above the industry monopoly price, and the price grid is evenly spaced. In Section 1.3, we discuss generalisations of (3).

Proposition 1. *Let $\overline{n} = \max_t |J_t|$. There exists no MPE with strategies of the form (3) if, for any $s = 2,\dots,k-T$,*

$$\pi(p^s) < \sum_{\tau=1}^{T-1} \left((\overline{n}-1)\pi\left(p^{s+\tau}\right)\right) + \left(\overline{n} - \delta^T\right)\pi\left(p^{s+T}\right). \tag{4}$$

The proof relies on a revealed preference argument. According to R^D, if the lowest com-

[3] We leave $\mu_i(p_{j\notin J_t})$ unspecified to admit a range of behaviour in the war of attrition. As noted by Noel (2008), with $n > 2$, false starts to the cycle and reversion to the trough are possible outcomes. We do not take a position on the nature of the war of attrition, and focus instead on the undercutting phase of the cycle.

mitted price is p^{s-1}, then p^s is a best response, while p^{s+1} is not. Setting a price of p^s provides a share of industry profits at this price, while setting p^{s+1} instead would deliver the entire market. For firm i to resist the temptation to undercut more aggressively, then either the value function must decline sharply in the lowest committed price or the industry profit function must be very convex. We can use R^D to calculate the shape of the value function, allowing us to isolate the conditions implied for the industry profit function.

In the MT model, the existence of an MPE with Edgeworth cycles relies on a sufficiently fine price grid. According to Proposition 1, if the price grid is fine and $\overline{n} \geq 2$, the profit function must be convex in prices to support the strategies R^D. Profits are generally concave in prices, suggesting that price commitment is unlikely to underpin Edgeworth cycles if $\overline{n} \geq 2$.

Fixing n, a greater commitment length T relaxes the convexity condition (4) by reducing the number of firms $|J_t|$ adjusting price in period t. In the special case considered by MT and Noel (2008), $T = n$, $|J_t| = 1$ for all t, and (4) is trivially violated. If instead firms adjust prices more frequently and $n > T$, then price commitment is unlikely to explain Edgeworth cycles.

1.2 Stochastic timing

In each period, each firm has an opportunity to set price with independent probability $x \in (0,1)$. The timing of play is as follows. At the beginning of each period t, each player learns privately whether they are able to adjust price in the current period. With probability x, each player chooses a new price; otherwise, she is committed to the last price she set. All prices then become public information, profits are received, and the period ends. As x approaches zero, the model converges to full commitment: when firm i moves, the conditional probability that her rivals also move approaches zero. We view this specification as a natural compromise between sequential and simultaneous play. Markets characterised by both frequent decision making and decision lags do not neatly match either sequential of simultaneous timing, but could be approximated by stochastic timing.

Because each firm may be committed to a previously set price, the entire price vector is payoff-relevant. Firm i's dynamic problem is determined by the Bellman equations

$$U_i(p) = xV_i(p) + (1-x)W_i(p), \tag{5}$$

$$V_i(p) = \max_{p_i} W_i(p), \tag{6}$$

$$W_i(p) = \mathbb{E}_{p_{j \neq i}}\left(\pi_i(p) + \delta U_i(p)\right). \tag{7}$$

$U_i(p)$ is the value to firm i at the beginning of a period t, given the price vector p. With probability x she has an opportunity to adjust price in the current period, and her valuation after this revelation is given by V_i. With probability $1-x$, firm i is committed to price and her valuation is determined by W_i. Expectations are taken over the ability of rivals to adjust prices in the current period and the (possibly mixed) strategies of rivals.

Given prices $p^1 > p^2 > \cdots > p^k$, consider symmetric strategies of the form

$$R_i^S(p) = \begin{cases} p^{s+1} & \text{if } \min_j p_j = p^s,\ s = 1,\ldots,k-1, \\ \left.\begin{array}{ll} p^1 & \text{with probability } \mu_i(p) \\ p^k & \text{with probability } 1-\mu_i(p) \end{array}\right\} & \text{if } \min_j p_j = p^k. \end{cases} \tag{8}$$

We will use p^s to refer to states in which $\min_j p_j = p^s$, for $s = 1,\ldots,k$. As before, off-path reactions are unconstrained, the principal features of an Edgeworth cycle are captured, and the strategies employed by MT are contained as a special case.

Proposition 2 examines the conditions required on the profit function for an MPE based on the strategies R^S. We set up the proposition by introducing the function $\gamma(n,x,\delta)$:

$$\gamma(n,x,\delta) = \frac{1 - \frac{\Delta}{n}\left(\frac{1-\delta}{1-\Delta}\right)}{h(n,x)} - \frac{\delta x}{1-\Delta}, \tag{9}$$

where

$$\Delta = \delta(1-x)^n, \qquad g(m) = \binom{n-1}{m} x^m (1-x)^{n-1-m}, \qquad h(n,x) = \sum_{m=0}^{n-1} \frac{g(m)}{m+1}.$$

With probability $(1-x)^n$, there is no change in the state vector in period t. The term Δ is therefore the discount factor associated with the continuation of the current state. For $m = 0,\ldots,n-1$, $g(m)$ is the probability that m rivals move in the current period. The function $h(n,x)$ then measures the expected market share of firm i in the undercutting phase (states with $p^s > p^k$) when it is her turn to move and all firms adopt the strategies R^S.

Proposition 2. *There exists no MPE with strategies of the form (8) if, for any $s = 2,\ldots,k-1$,*

$$\pi\left(p^s\right) < \gamma(n,x,\delta)\pi\left(p^{s+1}\right). \tag{10}$$

The proof of Proposition 2 follows a similar strategy to that of Proposition 1. Let us consider the implications for the industry profit function.[4] For $\delta < 1$, the convexity measure γ is

[4]We have made the simplifying assumption that the probability each firm moves is independent. If there is a positive correlation between moves, then this has two opposing effects on $\gamma(n,x,\delta)$. When firm i moves, there is a greater chance of rivals adjusting price, leading to a lower expected market share $h(n,x)$ and higher γ. There is also a higher probability of commitment by all firms, and therefore a higher probability of remaining at the current state. This raises the relative value of states with higher prices, leading to a decrease in γ. On balance, according to the correlated binomial model (Kupper and Haseman, 1978; Diniz et al., 2010), an increase in correlation between moves leads to an increase in γ, particularly for low x, strengthening our conclusions.

increasing in n and x and decreasing in δ; and γ approaches $1-\delta/n$ as x approaches 0 and $n-\delta$ as x approaches 1. Thus, for any fixed $n \geq 3$ and $\delta < 1$, there is an $x^* \in (0,1)$ such that for any $x \geq x^*$, $\gamma > 2$. If the price grid is sufficiently fine and firms choose prices frequently, this suggests the industry profit function needs to be implausibly convex to admit an MPE with Edgeworth cycles.

1.3 Discussion

While the strategies in (3) and (8) generalise those used by MT, they do contain two restrictive features. First, firms strictly undercut the lowest committed price in the undercutting phase. Price matching is precluded.[5] Price matching plays a similar role to extending the length of price commitment in our model, and would not alter our conclusions.

Second, (3) and (8) do not allow mixing over the price grid in the undercutting phase of the cycle. This lends predictability to the undercutting phase, allowing firms to deviate by undercutting rival prices and capturing the entire market. Suppose instead that at state p^s firms mix over a range of prices between p^{s+1} and p^{s+q} for $q > 1$. Then, the expected jump in market share for a firm deviating below p^{s+q} would be reduced. However, the logic of Propositions 1 and 2 continues to apply, only with reduced force: because the price grid is discrete, firms mix over finite support, and therefore still have an incentive to undercut more aggressively to capture the entire market.

Both models of partial price commitment suggest that with a moderate number of firms, price commitment is unlikely to explain Edgeworth cycles if firms adjust prices frequently. Using a unique market setting in which the precise timing of price changes is observed, de Roos and Katayama (2013) argue that firms do adjust prices frequently in the retail market for petrol in Perth, Western Australia.[6] Alternative explanations for Edgeworth cycles that are based on the theory of repeated games are offered by Clark and Houde (2013) and de Roos and Smirnov (2013). We leave it for future research to test the predictions of these theories.

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[5] If firm sizes are asymmetric, Eckert (2003) identifies asymmetric equilibria in the MT model in which the larger firm matches their rival's price rather than undercutting their rival.

[6] Additional, direct evidence is contained in our working paper, de Roos (2016).

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Appendices

A Proofs

Proof of Proposition 1

Proof. Suppose there exists an MPE with strategies of the form (3). Consider the perspective of firm $i \in J_t$. According to (3), $p^s \in R_i^D(p^{s-1})$ and $p^{s+1} \notin R_i^D(p^{s-1})$ for $s = 2, \ldots, k-1$. Hence,

$$\frac{1}{|J_t|}\pi\left(p^s\right) + \delta V_i^{T-1}\left(p^s\right) \geq \pi\left(p^{s+1}\right) + \delta V_i^{T-1}\left(p^{s+1}\right)$$
$$\Rightarrow \delta\left(V_i^{T-1}\left(p^s\right) - V_i^{T-1}\left(p^{s+1}\right)\right) \geq \pi\left(p^{s+1}\right) - \frac{1}{|J_t|}\pi\left(p^s\right). \tag{11}$$

Similarly, $p^{s+\tau} \in R_i^D(p^{s+\tau-1})$ and $p^{s+\tau+1} \notin R_i^D(p^{s+\tau-1})$ for $s+\tau = 2, \ldots, k-1$, which implies

$$\delta\left(V_i^{T-1}\left(p^{s+\tau}\right) - V_i^{T-1}\left(p^{s+\tau+1}\right)\right) \geq \pi\left(p^{s+\tau+1}\right) - \frac{1}{|J_t|}\pi\left(p^{s+\tau}\right).$$

Aggregating these inequalities for $\tau = 0, \ldots, T-1$ yields the condition

$$\delta\left(V_i^{T-1}\left(p^{s}\right) - V_i^{T-1}\left(p^{s+T}\right)\right) \geq \pi\left(p^{s+T}\right) + \sum_{\tau=1}^{T-1} \frac{|J_t|-1}{|J_t|}\pi\left(p^{s+\tau}\right) - \frac{1}{|J_t|}\pi\left(p^{s}\right). \tag{12}$$

The strategies in (3) imply

$$\begin{aligned} V_i^{T-1}\left(p^{s}\right) &= \delta^{T-1} V_i^0\left(p^{s+T-1}\right) \\ &= \delta^{T-1}\left(\frac{1}{|J_t|}\pi\left(p^{s+T}\right) + \delta V_i^{T-1}\left(p^{s+T}\right)\right). \end{aligned}$$

Combining with (12) and noting that $V_i^{T-1}\left(p^{s+T}\right)$ is non-negative leads to the condition

$$\pi\left(p^{s}\right) \geq \sum_{\tau=1}^{T-1} \left(|J_t|-1\right)\pi\left(p^{s+\tau}\right) + \left(|J_t| - \delta^T\right)\pi\left(p^{s+T}\right). \tag{13}$$

An equivalent expression is required for each period t, leading to the expression in (4). To focus on undercutting periods, we require $s \geq 2$. The constraints $s+\tau \leq k-1$ and $\tau \leq T-1$ imply $s \leq k-T$. Hence, (4) holds for $s = 2, \ldots, k-T$. With $k \geq T+2$, this is feasible. □

Proof of Proposition 2

Proof. We first introduce additional notation. Let $W_i(p^r, p^s, m)$ indicate the value to firm i when i is committed to price p^r, $\min_j p_j = p^s$ given the state p, and m rivals were partially committed to price p^s at the beginning of the current period. We use the shorthand $W_i(p^s) = W_i(p^s, p^s, n-1)$. Define $U_i(p^r, p^s, m)$ and $U_i(p^s)$ equivalently. Let $V_i(p^r, p^s)$ be the value to firm i when $\min_j p_j = p^s$ for state p, and i chooses $p_i = p^r$ in the current period.

We first use revealed preference to place a bound on the relationship between $U_i(p^s)$ and $U_i(p^{s+1})$. By (8), $V_i(p^s, p^{s-1}) \geq V_i(p^{s+1}, p^{s-1})$, which implies

$$\sum_{m=0}^{n-1} g(m)\left(\frac{\pi(p^s)}{m+1} + \delta U_i\left(p^s, p^s, m\right)\right) \geq \pi(p^{s+1}) + \delta U_i\left(p^{s+1}, p^{s+1}, 0\right).$$

Observe that, for $s = 1, \ldots, k-1$ and $m = 0, \ldots, n-1$,

$$
\begin{aligned}
U_i\left(p^s, p^s, m\right) &= xV_i\left(p^s\right) + (1-x)^n\left(\frac{\pi(p^s)}{m+1} + \delta U_i\left(p^s, p^s, m\right)\right) + (1-x)\delta\sum_{m'=1}^{n-1} g(m')U_i\left(p^s, p^{s+1}, m'\right) \\
&= \frac{(1-x)^n}{1-\Delta}\frac{\pi(p^s)}{m+1} + \frac{xV_i(p^s)}{1-\Delta} + \delta\frac{1-x}{1-\Delta}\sum_{m'=1}^{n-1} g(m')U_i\left(p^s, p^{s+1}, m'\right),
\end{aligned}
$$

and rearrange the above inequality to obtain

$$
\delta\left(U_i\left(p^s\right) - U_i\left(p^{s+1}\right)\right) \geq \frac{1}{n(1-\Delta)}\left(\pi\left(p^{s+1}\right)(n-\Delta) - \pi\left(p^s\right)(nh(n,x) - \Delta)\right). \tag{14}
$$

Next, we construct the relationship between $U_i(p^s)$ and $U_i(p^{s+1})$ using R^S.

$$
\begin{aligned}
U_i\left(p^s\right) = x\sum_{m=0}^{n-1} g(m)&\left(\frac{\pi(p^{s+1})}{m+1} + \delta U_i\left(p^{s+1}, p^{s+1}, m\right)\right) + (1-x)^n\left(\frac{\pi(p^s)}{n} + \delta U_i\left(p^s\right)\right) \\
&+ (1-x)\delta\sum_{m=1}^{n-1} g(m)U_i\left(p^s, p^{s+1}, m\right).
\end{aligned}
$$

Simplify this expression to obtain

$$
U_i\left(p^s\right) = \frac{\delta-\Delta}{1-\Delta}U_i\left(p^{s+1}\right) + \pi\left(p^s\right)\frac{(1-x)^n}{n(1-\Delta)} + \pi\left(p^{s+1}\right)\frac{xh(n,x) - \frac{\Delta(1-(1-x)^n)}{n}}{(1-\Delta)^2}. \tag{15}
$$

Combine (14) and (15) to obtain

$$
\delta\left(\frac{\delta-1}{1-\Delta}\right)U_i\left(p^{s+1}\right) \geq \frac{1}{n(1-\Delta)}\left(\pi\left(p^{s+1}\right)\left(n - \Delta - \frac{\delta nxh(n,x) - \Delta(\delta-\Delta)}{1-\Delta}\right) - \pi\left(p^s\right)nh(n,x)\right).
$$

Observe that $U_i(p^{s+1})$ must be non-negative. Therefore,

$$
\pi\left(p^s\right) \geq \left(\frac{1 - \frac{\Delta}{n}\left(\frac{1-\delta}{1-\Delta}\right)}{h(n,x)} - \frac{\delta x}{1-\Delta}\right)\pi\left(p^{s+1}\right).
$$

If this condition is violated then there is no MPE based on the strategies R^S. □